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VIBRATION CONTROL OF CABLE-STAYED BRIDGES—PART 2: CONTROL ANALYSES

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SUMMARY

The objective of this research is to increase the understanding on how the complexities associated with modelling cable-stayed bridges, such as non-linear behaviour and the participation of coupled, high-order vibration modes in the bridge's dynamic response, affect the overall effectiveness of active control schemes. Using a reduced-order state-space model, control analyses examine the effectiveness of full state feedback control employing a Linear Quadratic Regulator (LQR) and the effectiveness of dynamic output feedback control utilizing a Kalman–Bucy filter in attenuating the structure's force time-history response. Results show that significant reductions of the maximum internal forces and the force/displacement response can be achieved through full state or output feedback control. An investigation of various actuator configurations leads to the conclusion that actuators are most effective when located close to the centre of the bridge span. The study also shows that only first-order modes need to be controlled to reduce the displacement response; however, the control of higher-order modes is essential to reduce the force response. Multiple-support excitation needs to be considered since it can excite entirely different modes than uniform-support excitation. Moreover, multiple-support excitation induces forces that are caused by pseudo-static displacements and can not be controlled. Special attention needs to be given to coupled modes since their control can lead to an increased force response of the structure. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: cable-stayed bridge; linear quadratic regulator; full state feedback; output feedback; high order modes; coupled modes

INTRODUCTION

This investigation represents a continuation of Part 1,[1] which examined modelling issues associated with the Jindo Cable-Stayed Bridge. Part 2 focuses exclusively on the application of active control to the model created in Part 1. As stated earlier, the objective of this investigation is to increase the understanding of how the complexities associated with modelling cable-stayed bridges, such as non-linear behaviour and the participation of highly coupled, high-order vibration modes in the overall dynamic response, affect the overall effectiveness of active control schemes. In this study, the Linear Quadratic Regulator (LQR) is used for its effectiveness and ease of application. Central to this control theory is the performance index, J, defined as

$$J = \int_0^{t_f} [\mathbf{x}^T(t)\,\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\,\mathbf{R}\mathbf{u}(t)]\,dt \tag{1}$$

which is to be minimized subject to system equations of motion. Both $\mathbf{Q}$ and $\mathbf{R}$ are user-specified weighting matrices, $\mathbf{x}(t)$ is the state vector and $\mathbf{u}(t)$ is the vector of control inputs. By adjusting $\mathbf{Q}$ and $\mathbf{R}$, priority can be given to the minimization of the state variables or to the minimization of control forces.

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Various issues affecting control, such as the impact of higher-order modes, modal coupling and actuator placement, are investigated under the assumption of full state feedback. Next, the linear quadratic estimator employing a *Kalman–Bucy* filter is introduced. Control analyses are performed using total acceleration output feedback. The degradation of the results relative to the full state feedback case and different sensor configurations are examined. The majority of analyses are performed considering the Northridge earthquake discussed in Part 1. Analyses considering the Imperial Valley earthquake (also discussed in Part 1) are presented later. The effectiveness and efficiency of active control partially depends on the actuator layout and on the number of actuators used. To examine the effectiveness of different actuator configurations, control analyses are performed using 4, 3, and 2 actuators acting in both lateral and vertical directions. The locations of the actuators were chosen and illustrated in Part 1 of this investigation on the basis that the most damaging modes are rendered most controllable.

CONTROL ANALYSIS USING FULL STATE FEEDBACK CONTROL

The focus of this section is to identify the importance of the following issues in active control of cable-stayed bridges: (1) impact of higher-order modes in the control of the bridge deck's force response; (2) importance of multiple-support excitation; (3) control of coupled tower-deck modes using actuators located on deck; (4) effectiveness of various actuator and sensor configurations; (5) effects of control on tower vibrations; and (6) effects of control on coupled deck modes. Control efforts are focused mainly on controlling the force response, since the objective of the control is to minimize any potential damage as a result of seismic ground excitation. Controlling the displacements is only of secondary importance; for lifeline structures such as cable-stayed bridges, only a small potential for damage exists due to excessive displacements. In this case, displacement control primarily serves for occupant comfort. The scenario is different for building structures, where excessive displacements can result in significant structural and non-structural damage.

The participation of higher-order modes in the force response of cable-stayed bridges has been discussed previously.[1] Specifically, the modal analysis in Part 1 of this study showed the extent to which each mode participates and the locations where the largest forces can be expected. Since higher-order modes participate significantly in the bridge's force response, the control of higher modes is necessary to achieve a reduction in the bridge's force response. However, most of the higher-order modes, i.e. all modes except modes 1 to 4 and modes 14, 15, 17 and 18 which represent first-order modes, have little effect on the structure's displacement response. To illustrate the importance of higher-order modes, the bridge is subjected to uniform and multiple-support excitation in the lateral, vertical and longitudinal direction. Unless otherwise indicated, results presented in this paper pertain to analyses performed using the Northridge earthquake and the special case of multiple-support excitation in which the supports move out of phase by 180°. Furthermore, results are generally only discussed in detail for the case of lateral support excitation, since a similar trend in the results is observed for the cases of vertical and longitudinal support excitation. Additional results can be found in Schemmann and Smith.[2]

Impact of high-order modes: uniform-support excitation

Under lateral uniform-support excitation, five different cases are studied considering control of: (1) only mode 1; (2) modes 1 and 9; (3) modes 1, 9 and 26; (4) modes 1, 9 and 35; (5) modes 1, 9, 26 and 35. Modes 1, 9, 26, and 35 represent the first, third, fifth and seventh lateral deck mode, respectively. Modal displacements of higher-order modes are generally smaller in magnitude than modal displacements of lower-order modes. Since the control effort is proportional to $\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}$, larger values are assigned to the $\mathbf{Q}$-elements corresponding to the higher-order modes. Thus, an approximately equal control effort is achieved for the higher- and lower-order modes. The value of $Q(1, 1)$ is chosen arbitrarily in this case, since no specifications for the attenuation of the force response are given. The remaining values of $\mathbf{Q}$ are chosen such that maximum efficiency in terms of force response reduction per unit control effort is achieved. All elements of the weighting

matrix **Q** are assigned the value 0, except for: case 1, where the diagonal element of **Q** corresponding to the displacement of mode 1 is assigned the value 1; case 2, where the diagonal elements of **Q** corresponding to the displacement of modes 1 and 9 are assigned the values 1 and 100, respectively; case 3, where the diagonal elements of **Q** corresponding to the displacement of modes 1, 9 and 26 are assigned the values 1, 100 and 500, respectively; case 4, where the diagonal elements of **Q** corresponding to the displacement of modes 1, 9 and 35 are assigned the values 1, 100 and 800, respectively; and case 5, where the diagonal elements of **Q** corresponding to the displacement of modes 1, 9, 26 and 35 are assigned the values 1, 100, 500 and 800, respectively. For all five cases, the diagonal elements of the **R** matrix are assigned the value 0·00005. Equal values are chosen, since all actuators are considered equally important and are assumed to have the same output force capacity.

To measure the effectiveness of control, the 2-norm of the displacements, $\mathbf{v}_a(t)$, moments, $\mathbf{f}(t)$, and control forces, $\mathbf{u}(t)$, are calculated for every degree of freedom over the entire time history.

Table I displays the 2-norm of the control forces and the percentage reduction achieved in the 2-norms of the uncontrolled lateral displacement and force response. The effectiveness of the controller is also illustrated by showing the percent reduction in the uncontrolled responses per unit control effort. For this example, the time-history response of the vibrational lateral bridge displacements is termed 'displacement response', and the time-history response of the bending moments about the vertical axis is termed 'force response'. The largest reduction in the open-loop force response is achieved when modes 1, 9, 16, and 35 are controlled. This setup is the most efficient in terms of force response reduction per unit control effort (column 5 of Table I), although it requires the largest control forces. If the objective was to reduce the displacements, only mode 1 should be controlled, since highest efficiency is achieved for this case.

Figures 1–5 show some of these results graphically. Specifically, Figure 1 shows the controlled and uncontrolled lateral displacements at the centre of the deck for the cases when only mode 1 and when modes 1, 9, 26 and 35 are controlled. The earthquake ground motion, which is scaled to the experimental model by dividing the original time scale by $\sqrt{150}$, ends at 1·8 sec. Both cases offer significant reductions in the uncontrolled displacement response. The control of the higher-order modes largely eliminates the high-frequency content of the controlled response; otherwise, the controlled responses for both cases are almost identical.

Figure 2 displays the controlled and uncontrolled bending moments about the vertical axis for the cases when only mode 1 and when modes 1, 9, 26, and 35 are controlled. Controlling only the first mode does not control the force response adequately, although it did significantly control the displacement response. A significant reduction in the controlled (forced and free) response can be observed when the higher-order modes are controlled in addition to the first mode. Figure 3 which illustrates the peak controlled and uncontrolled bending moments along the deck, verifies this finding. Furthermore, Figure 3 also shows that the reduction of the peak moments extends along the whole bridge deck, including the side span where no actuators are located.

Table I. Reduction of uncontrolled displacement and force response, reduction per unit control force, and required control force for lateral uniform-support excitation

Case	Modes controlled	$\Vert\mathbf{v}_a(t)\Vert_2$ (% reduction)	$\frac{\Vert\mathbf{v}_a(t)\Vert_2}{\Vert\mathbf{u}(t)\Vert_2}$ (%/N)	$\Vert\mathbf{f}(t)\Vert_2$ (% reduction)	$\frac{\Vert\mathbf{f}(t)\Vert_2}{\Vert\mathbf{u}(t)\Vert_2}$ (%/N)	$\Vert\mathbf{u}(t)\Vert_2$ (N)
1	1	60·89	16·43	21·59	5·85	74·97
2	1, 9	69·26	14·76	64·26	13·73	94·95
3	1, 9, 26	69·26	14·58	64·26	13·55	96·72
4	1, 9, 35	69·34	13·59	67·30	13·19	103·37
5	1, 9, 26, 35	69·42	13·55	70·33	13·73	103·77

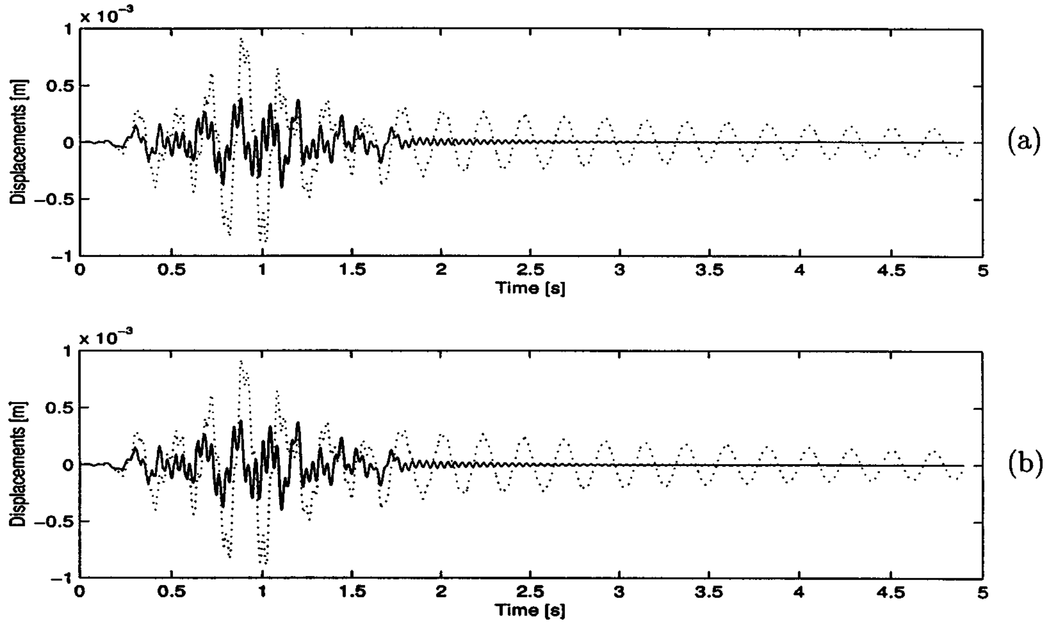


Figure 1. Controlled (solid line) and uncontrolled (dotted line) lateral displacement time histories at mid-span subject to lateral uniform-support excitation: (a) mode 1 controlled; (b) modes 1, 9, 26 and 35 controlled

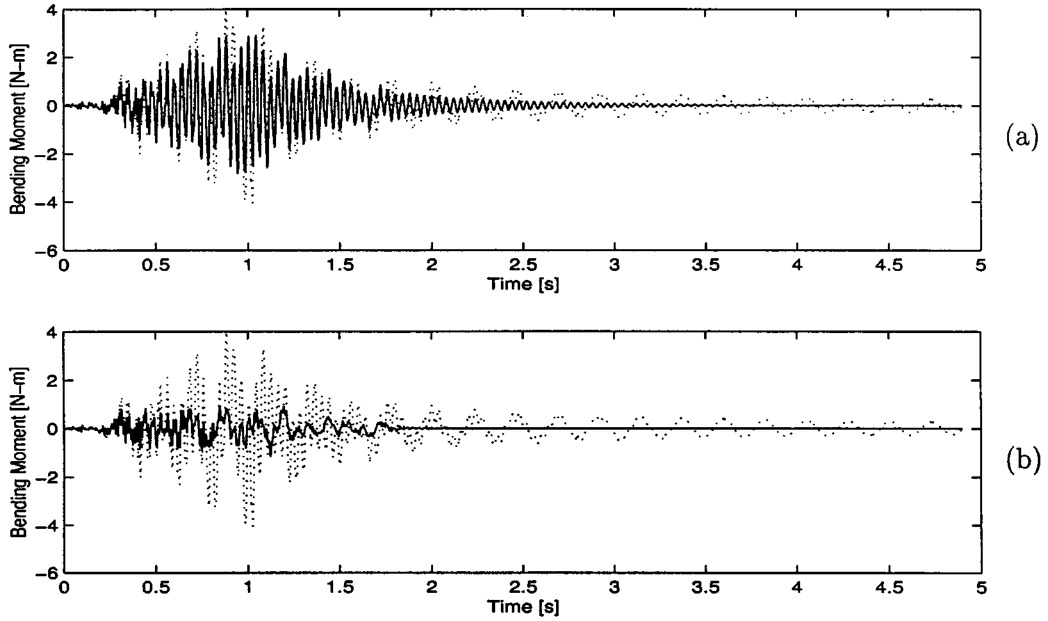


Figure 2. Controlled (solid line) and uncontrolled (dotted line) lateral bending moment time histories about the vertical axis at mid-span subject to lateral uniform-support excitation: (a) mode 1 controlled; (b) modes 1, 9, 26 and 35 controlled

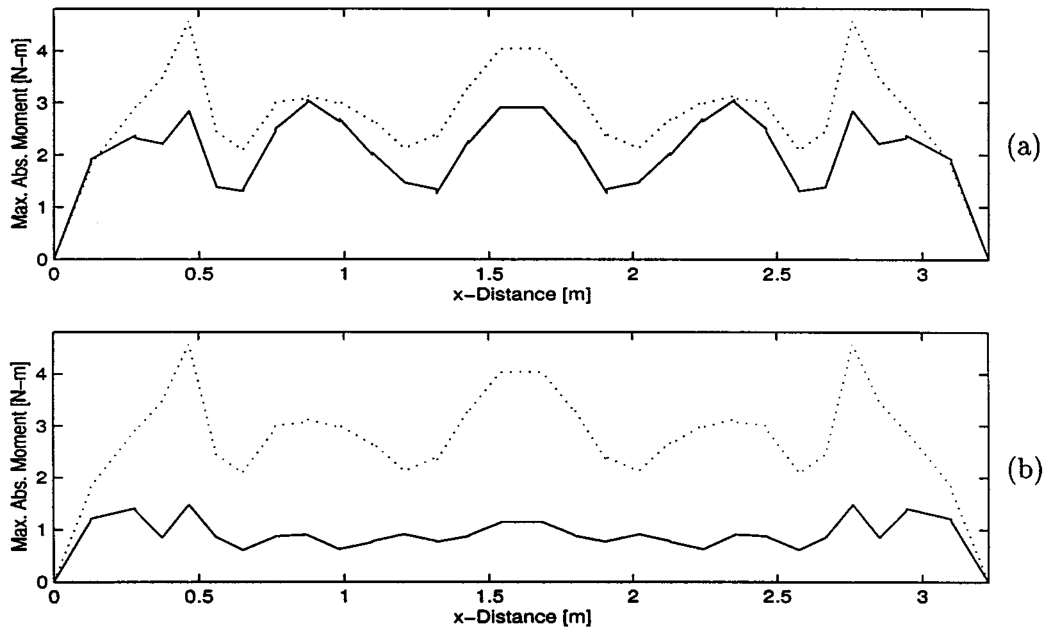


Figure 3. Controlled (solid line) and uncontrolled (dotted line) maximum lateral deck bending moments subject to lateral uniform-support excitation: (a) mode 1 controlled; (b) modes 1, 9, 26 and 35 controlled

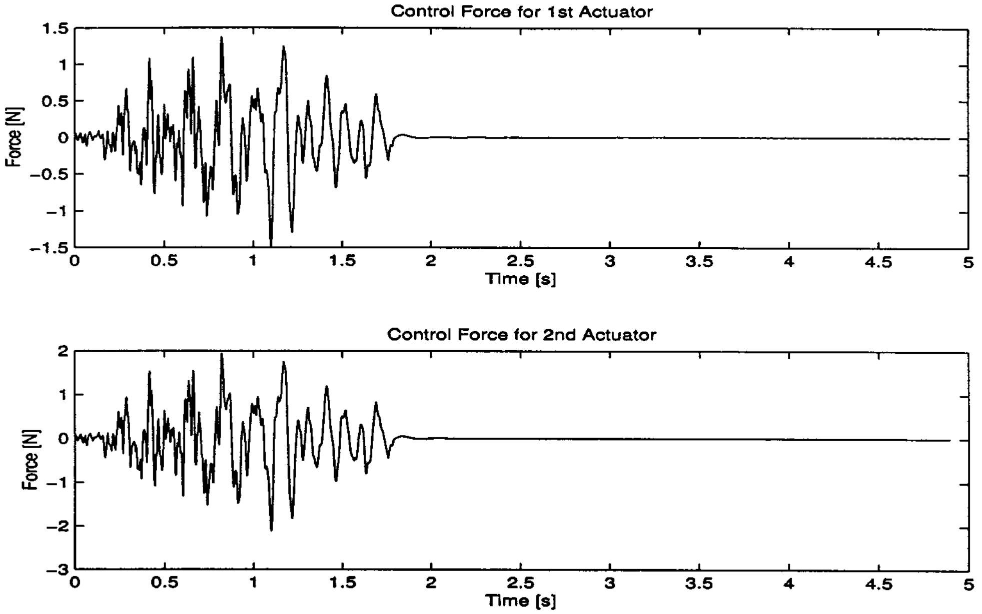


Figure 4. Control forces for actuators located at deck nodes 11 and 21 (labelled 1st actuator), and deck nodes 14 and 18 (labelled 2nd actuator) for the case of lateral uniform-support excitation: mode 1 controlled

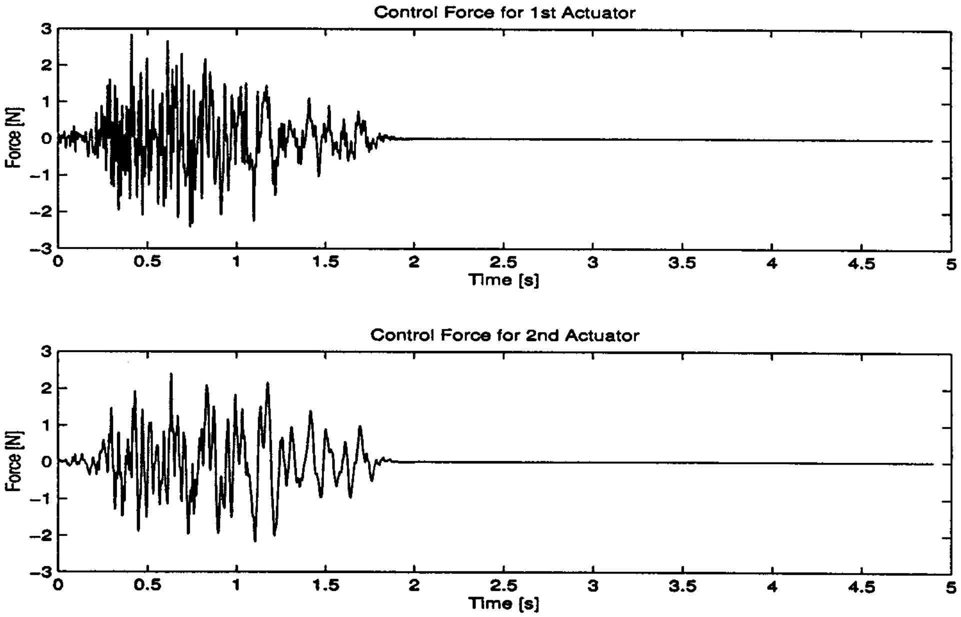


Figure 5. Control forces for actuators located at deck nodes 11 and 21 (labelled 1st actuator), and deck nodes 14 and 18 (labelled 2nd actuator) for the case of lateral uniform-support excitation: modes 1, 9, 26 and 35 controlled

Figure 4 shows the actuator output forces located at deck nodes 11, 14, 18, and 21 (see actuator layout in Figure 9 of Part 1) for the case when only mode 1 is controlled. Figure 5 shows the actuator output forces for the case when modes 1, 9, 26, and 35 are controlled. The figures only display the output of 2 actuators since the bridge vibrates symmetrically under uniform-support excitation. Thus, actuators located at nodes 11 and 21, and actuators located at nodes 14 and 18 have equal force outputs. Naturally, the force output for the case when modes 1, 9, 26, and 35 are controlled is greater, requiring larger capacity actuators, than for the case when only the first mode is controlled. It should also be noted that each Newton of control force applied to the experimental model is equivalent to applying 7500 Newtons to the prototype model. Thus, considering the Baldwin Hills-Northridge earthquake, actuators would have to be capable of providing approximately 22 500 Newtons force to achieve the similar attenuations in the prototype bridge.

This example clearly illustrates the significance of higher-order modes when used in conjunction with active control. To reduce the force response, which is the primary cause of structural damage, higher-order modes need to be controlled.

The same type of analyses are performed for vertical and longitudinal uniform-support excitation as for the case of lateral uniform-support excitation. However, since the results show similar trends to those observed under lateral excitation, the results are not illustrated to the same detail as in the previous section. Table II summarizes all results obtained for the case of lateral, vertical and longitudinal uniform-support excitation. Under vertical uniform-support excitation comparisons are made for the cases when modes 2, 8, 13, and 21 are controlled or when only a subset of these modes is controlled. These modes represent the 1st, 5th, 7th and 9th vertical deck modes. Under longitudinal uniform-support excitation comparisons are made for the cases when some or all of modes 3, 6 and 10, representing 2nd, 4th, and 6th vertical deck modes, are controlled. Table II supports the conclusion that under uniform support excitation, control of the first mode is sufficient to reduce the deck's displacement response, but control of the higher order modes is required to attenuate the deck's force response. Additional details on these analyses and results can be found in Schemmann and Smith.[2]

Table II. Reduction of uncontrolled displacement and force response and required control force for lateral, vertical, and longitudinal uniform-support excitation

	Modes controlled	1	1, 9	1, 9, 26, 35
Lateral excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	60·89%	69·26%	69·42%
	Moments, $\|\mathbf{f}(t)\|_2$	21·59%	64·26%	70·33%
	Control effort, $\|\mathbf{u}(t)\|_2$	74·97 N	94·95 N	103·77 N
	Modes controlled	2	2, 8	2, 8, 13, 21
Vertical excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	67·74%	67·88%	71·11%
	Moments, $\|\mathbf{f}(t)\|_2$	16·28%	16·50%	53·79%
	Control effort, $\|\mathbf{u}(t)\|_2$	50·18 N	57·51 N	127·71 N
	Modes controlled	3	3, 6	3, 6, 10
Longitudinal excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	57·07%	61·89%	69·34%
	Moments, $\|\mathbf{f}(t)\|_2$	14·90%	19·25%	45·03%
	Control effort, $\|\mathbf{u}(t)\|_2$	6·98 N	9·99 N	23·67 N

Impact of high-order modes: multiple-support excitation

The modal analysis presented in Part 1 of this study showed that higher-order modes play a less dominant role in the force response under multiple-support excitation than under uniform-support excitation. The control analyses verify this finding; considerable attenuation of the force response can be achieved by only controlling the first-order modes. Controlling higher-order modes can reduce the force response, but not very significantly. In this section, the specific case of multiple-support excitation is considered, in which the supports move at equal amplitudes, but 180° out of phase relative to the adjacent supports. Furthermore, this discussion focuses on the case in which multiple-support excitation is applied in the lateral direction and vibrations are controlled using 4 actuators. The vertical and longitudinal excitation cases are only summarized, since the findings reflect those obtained for lateral excitation.

Table III shows the reductions obtained in the 2-norms of the uncontrolled total displacement and total force responses and the 2-norm of the control effort considering lateral, vertical and longitudinal multiple-support excitation. This table displays the attenuation of both the total (pseudo-static and vibrational) and vibrational displacements/forces. Pseudo-static displacements and forces are unique to multiple-support excitation, as they are induced by the differential motion of the support. Hence, these pseudo-static displacements are not controlled, resulting in a decrease in effectiveness of active control under multiple-support excitation. The controlled responses in the lateral direction are obtained by assigning the values 10 and 100 to the elements of **Q** corresponding to modes 4 and 20, respectively. All the diagonal elements of the **R** matrix are assigned the value 0·00005. Under lateral multiple-support excitation, mode 4, representing the second lateral deck mode, dominates the displacement and force response of the structure. Solely controlling this mode results in significant attenuation of the displacement and force response. Mode 20, representing the fourth lateral deck mode, is the next highest mode to participate in the force response of the bridge deck. However, controlling this mode yields only mild improvements in the displacement and force response. It should be noted that a more intricate optimization of **Q** and **R** may result in a more efficient control of modes 4 and 20.

Most of the findings found for the case of lateral multiple-support excitation are also valid for the case of vertical and longitudinal multiple-support excitation. Since most of the forces and displacements are induced by the first-order modes for both of these cases, significant attenuation of the force and displacement responses can be achieved by controlling first-order modes. The control of higher-order modes is not always

Table III. Reduction of uncontrolled displacement and force response and required control force for lateral, vertical, and longitudinal multiple-support excitation

	Modes controlled	4	4, 20
Lateral excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	38·34%* (70·83%)†	38·64% (71·71%)
	Moments, $\|\mathbf{f}(t)\|_2$	50·20% (62·41%)	54·58% (68·87%)
	Control effort, $\|\mathbf{u}(t)\|_2$	97·52 N	104·04 N
	Modes controlled	3	3, 6, 16
Vertical excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	67·07%* (72·99%)†	67·34% (73·28%)
	Moments, $\|\mathbf{f}(t)\|_2$	66·01% (66·93%)	69·27% (70·23%)
	Control effort, $\|\mathbf{u}(t)\|_2$	119·66 N	128·61 N
	Modes controlled	2	2, 5
Longitudinal excitation	Displacements, $\|\mathbf{v}_a(t)\|_2$	69·66%* (73·49%)†	70·52% (74·59%)
	Moments, $\|\mathbf{f}(t)\|_2$	57·27% (60·52%)	65·95% (69·82%)
	Control effort, $\|\mathbf{u}(t)\|_2$	105·21 N	111·87 N

* Pseudo-static and vibrational response
† Vibrational response only

profitable, as illustrated for the case of vertical multiple-support excitation. For this excitation, Table III shows that an increase in the control effort of 7·5% results in an additional reduction of 4·9% in the force response when modes 6 and 16 are controlled in addition to mode 3. It is probably more efficient to increase the control gain of mode 3 to achieve additional attenuation of the force response, than to control modes 6 and 16. However, a more refined optimization of the **Q** and **R** matrices may render the control of modes 6 and 16 more efficient. Under longitudinal multiple-support excitation the majority of attenuation in the force and displacement response is achieved through the control of mode 2. In this case the control of mode 5 also results in considerable reductions, especially in the peak bending moments at the center of the bridge. Thus, both modes 2 and 5 should be controlled.

In summary, multiple-support excitation adds another complexity to the control analysis of cable-stayed bridges. Pseudo-static displacements and forces are introduced, which cannot be controlled. Thus, additional modes need to be controlled, affecting the actuator configuration. The bridge also exhibits a different dynamic behaviour under multiple-support excitation than under uniform-support excitation. The effectiveness of active control for uniform and multiple-support excitation should be assessed on the basis of total induced forces, which take into consideration all pseudo-static effects, while ignoring effects of rigid-body translations. A general conclusion that active control is more effective for uniform-support excitation can not be made since the effectiveness is highly dependent on such issues as actuator locations, modeling of multiple-support excitation and frequency content of the earthquake. Furthermore, based on the same reasoning, general conclusions can not be made that higher-order modes typically have a larger impact on the control of cable-stayed bridges when subject to uniform-support excitation than when subject to multiple-support excitation.

Control of coupled tower-deck mode

The time-history analysis of Part 1 (see Figure 5, Part 1) shows that mode 18, representing a first-order symmetric tower mode acting in the longitudinal direction, induces relatively large deck bending moments when subjected to longitudinal uniform-support excitation. To reduce the overall force response in the deck, an attempt is made to control the coupled motion utilizing the actuators located on the deck. A control

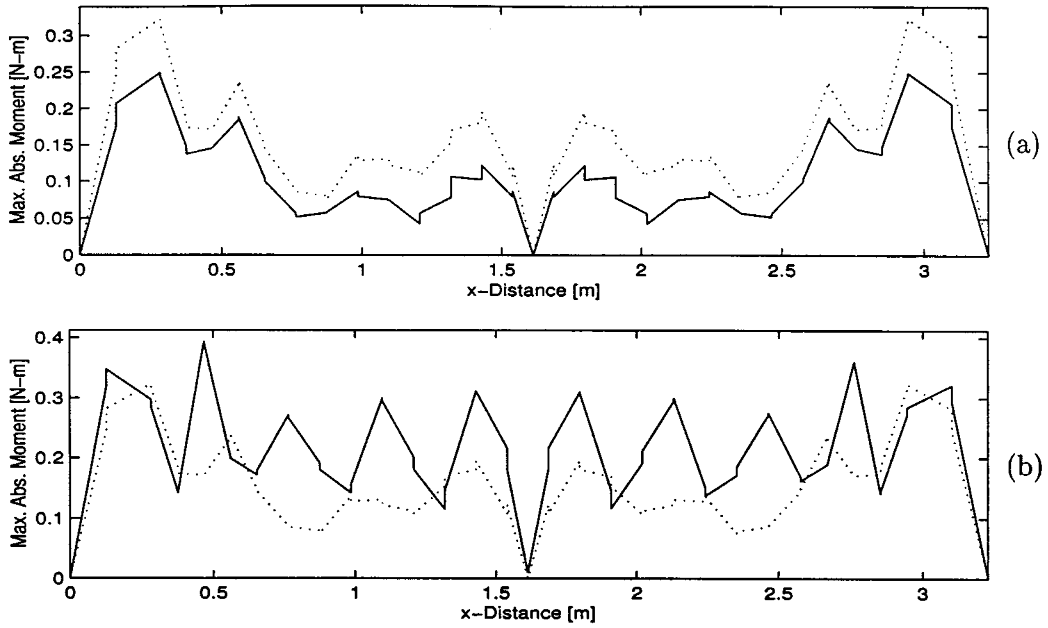


Figure 6. Controlled (solid line) and uncontrolled (dotted line) maximum vertical deck bending moments subject to longitudinal uniform-support excitation: (a) modes 3, 6 and 9 controlled; (b) modes 3, 6, 9, and 18 controlled

analysis has already been performed in the previous section for the case when the bridge is subjected to longitudinal uniform-support excitation; however, no attempts have been made to control the forces induced by mode 18. In this analysis, elements of **Q** corresponding to modes 3, 6 and 9 were assigned values of 5, 10 and 50, respectively, while all the diagonal elements of **R** were assigned values of 0·00005 each. To control mode 18, the element of **Q** corresponding to this mode is assigned a value of 500, which is significantly larger than the values used for modes 3, 6 and 9.

Figure 6 shows the maximum bending moments about the lateral axis for the cases when modes 3, 6 and 9 and modes 3, 6, 9 and 18 are controlled. The figure reveals that, in an attempt to decrease the participation of mode 18 in the force response, its participation is actually increased. At numerous locations, the maximum bending moments are observed to be more than twice the magnitude of the uncontrolled bending moments. The control of mode 18 (in addition to the control of modes 3, 6 and 9) results in a 32·5 per cent *increase* in the 2-norm of the uncontrolled force response for a control effort of 109·76 N (measured as 2-norm). Without controlling mode 18, a 45·0 per cent *decrease* in the same 2-norm is achieved for a control effort of 23·72 N. Surprisingly, the 2-norm of the uncontrolled displacement response is still reduced by 48·7 per cent, when mode 18 is controlled; however, a larger 69·34 per cent reduction is achieved, when mode 18 is not controlled.

These results show that tower mode 18 should not be controlled with the current actuator configuration. Instead, control efforts should focus on controlling the tower mode, which induces the deck motion, possibly using actuators located on the towers. The example given here illustrates the complications modal coupling introduces into the control. Since cable-stayed bridges generally exhibit a great deal of modal coupling, special care must be taken when performing control analyses for these structures.

Comparison of actuator configurations

To investigate the effectiveness of different actuator configurations, control analyses are performed considering 4, 3, and 2 actuators acting in both lateral and vertical directions. The performance of the actuator configurations are evaluated based on two criteria. The first criterion considers the configuration's

Table IV. Reduction of uncontrolled force response, actuator efficiency and maximum control forces for lateral, vertical, and longitudinal *uniform-support excitation*

	Actuators used	Moments $\|\mathbf{f}(t)\|_2$ (%)	$\|\mathbf{f}(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Peak ctrl. force $\max \lvert\mathbf{u}(t)\rvert$ (N)
Lateral excitation	4	70·33	3·05	2·84
	3	73·48	3·07*	5·27
	2	67·87	2·94	5·13
Vertical excitation	4	53·79	1·09*	3·69
	3	46·69	1·64	5·40
	2	46·74	1·65	6·84
Longitudinal excitation	4	45·03	8·57	0·63
	3	45·17	8·61*	1·17
	2	45·17	8·61*	1·17

Table V. Reduction of uncontrolled force response, actuator efficiency and maximum control forces for lateral, vertical, and longitudinal *multiple-support excitation*

	Actuators used	Moments $\|\mathbf{f}(t)\|_2$ (%)	$\|\mathbf{f}(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Peak ctrl. force $\max \lvert\mathbf{u}(t)\rvert$ (N)
Lateral excitation	4	54·58	2·36*	2·84
	3	54·61	2·36*	4·55
	2	54·61	2·36*	4·55
Vertical excitation	4	69·27	2·42	4·05
	3	71·21	2·51*	6·98
	2	71·21	2·51*	6·98
Longitudinal excitation	4	65·95	2·65	4·32
	3	69·90	2·81*	4·41
	2	58·51	2·35	4·95

effectiveness in reducing the model's force response. The second criterion considers the peak control effort that is demanded from the actuators. The peak control effort dictates the required size of the actuator, which are to be kept as small as possible. Control analyses using 4 actuators were performed in the previous sections; however, only 2 and 3 actuators are used in this section. Analyses for both uniform and multiple-support excitations are performed; however, only analyses are performed in which a wide range of modes are controlled.

Tables IV and V display the 2-norm of the force response, the ratio of the 2-norms between the force response and the control effort, and the maximum control effort observed in any given actuator. To offer a comparison between all actuator configurations, both tables also show the results for the 4-actuator configuration, which were obtained in the previous sections. The values designated with a * in the second column of Tables IV and V indicate the most efficient actuator configurations. A comparison of the results for the case of lateral support excitation reveals that the configuration using 3 actuators is most efficient. All three systems are equally effective under multiple-support excitation and the results are very close under uniform-support excitation. However, the system using 4 actuators has a significant advantage, when comparing the peak actuator efforts of the three systems. Peak control efforts are almost half of those observed for the systems using 2 and 3 actuators. This advantage may be nullified if the additional actuator required for the 4-actuator configuration exceeds the cost of the larger actuators used in the 3-actuator configuration.

Vertical and longitudinal support excitation fall into one category since they both exert deck bending moments about the lateral axis. Under this condition, the system using 3 actuator outperforms the system using 2 actuators in all aspects; hence, only a comparison between the 3 and 4-actuator systems is necessary. Between these two system, the configuration using 3 actuators is more efficient 3 out of 4 times (all cases except for vertical uniform-support excitation). However, the margins of improvements are relatively small, such that peak control efforts should also be considered. Comparing maximum control efforts shows that actuators are required to deliver a total of 6·98 N (52·3 kN for the prototype) for the 3-actuator system while only 4·32 N (32·4 kN for the prototype) are required for the 4-actuator system. Thus, the configuration using 4 actuators is used to control vertical deck vibrations. Unless indicated otherwise, the 4-actuator configuration is used for the remainder of this study.

The results indicate that the control efficiency depends on the actuator locations. Generally, actuators located toward the center of the deck are most effective, explaining why the 3-actuator system outperforms the 4-actuator system most of the time. A larger quantity of actuators may be desirable since it typically decreases the requirements on actuator sizes while increasing the system robustness (in case of an actuator failure).

Effects of control on tower response

So far the focus of active control has been on attenuating the deck's force response. However, in an attempt to control the deck's force response, the towers' force response may also be altered (through the actuators located on the deck). It turns out, that the control efforts only affect the force response of the towers significantly if the deck modes that are being controlled also participate in the towers' response. For instance, under uniform and multiple-support excitation, lateral deck modes do not participate in the towers' force response to a large extent; hence, the towers' force response is not reduced when the deck is controlled. On the other hand, under vertical excitation, the majority of the towers' force responses are induced by deck modes. For the case of uniform-support excitation, modes 2, 5, 8 and 17, of which only mode 17 represents a tower mode, dominate the force response of the tower. Therefore, when modes 2, 5 and 8 are controlled, the 2-norm of the towers' bending moments about the lateral axis decrease by 54·2 per cent. Similarly, for the case of multiple-support excitation, mode 3, representing a vertical deck mode, dominates the force response of the towers. As a result, the 2-norm of the towers' bending moments about the lateral axis are reduced by 70·1 per cent when the deck is controlled.

Under longitudinal uniform-support excitation, mode 18, representing primarily a tower mode, dominates the force response. The 2-norm of the towers' bending moments about the lateral axis increases by almost 2 per cent when control is applied to the bridge deck. The previous section showed that mode 18 also participates in the vertical deck response. However, the control of mode 18 resulted in an increase in the deck's bending moments about the lateral axis. Examining the towers under this control effort reveals that the towers' force response is actually attenuated by 26·3 per cent (measured as 2-norm). Nevertheless, mode 18 should not be controlled using the actuators located on the deck. The increase in control effort and increase of the deck's force response does not warrant the reduction in the towers' force response. Under longitudinal multiple-support excitation, modes 2, 5 and 17 dominate the towers' force response. Relatively large attenuation of 43·7 per cent in the 2-norm of the towers' force response are due to the participation of modes 2 and 5 which are both deck modes.

Torsional moments and moments about the longitudinal axis in the towers are of negligible size for vertical and longitudinal support excitation and, hence, the effect of control on these moments is not discussed.

Effects of control on coupled deck forces

Many of the bridge's modes are coupled, resulting in coupling of the deck forces. For instance, most of the lateral deck modes also have a torsional component. However, the 2-norm of the torsional moments are generally approximately 5 per cent of the size of the uncontrolled moments about the vertical axis when the

bridge is subjected to the Northridge earthquake. Although the control of the torsional motion is not emphasized, the 2-norm of the torsional moments are reduced by 62 per cent when the lateral deck motion is controlled (for both uniform and multiple-support excitation).

Similarly, axial forces are induced through the interaction of the bridge deck and the cables when the bridge is subjected to vertical and longitudinal ground motion. These forces are of considerable magnitude as the 2-norms of uncontrolled axial forces range between 200 and 450 per cent of the uncontrolled vertical shear forces, except for the case of longitudinal multiple-support excitation. For this case, total axial forces are very large since they are induced by pseudo-static displacements acting in the longitudinal (i.e. axial) direction. Hence, no reductions in axial forces are achieved when the bridge deck is controlled in the vertical direction. The scenario is different for the case of longitudinal uniform-support excitation and vertical uniform/multiple-support excitation. Here, attenuations of 23·8, 67·6 and 69·9 per cent, respectively, in 2-norm of uncontrolled axial forces are achieved when the vertical bridge motion is controlled. Additional details on these results can be found in Schemmann and Smith.[2]

CONTROL ANALYSIS USING OUTPUT FEEDBACK CONTROL

So far only full state feedback control has been employed to obtain an upper bound on the performance of output feedback control. The question of interest is how well does output feedback control attenuate the structure's force response in comparison to full state feedback control. To implement output feedback control, a Kalman filter estimator is used. Since the system is assumed to be time invariant, the implementation of the filter only depends on the selection of the sensor and excitation noise intensities. Sensors with a full range of $\pm 2g$ are chosen since maximum deck accelerations of $1{\cdot}2g$ and $1{\cdot}9g$ are observed for the Northridge and the Imperial Valley earthquake, respectively. For the simulations, sensor noise is assumed to be white and additive, i.e. it does not depend on the magnitude of the sensor measurements, with a root mean squared error of 0·5 per cent of full scale. A sampling rate of 100 samples per second is used; higher sampling rates did not improve the performance of the Kalman filter significantly. A more detailed description on the Kalman filter used for this study is given in the Schemmann and Smith.[2]

To investigate the performance of output feedback control, control analyses are carried out using the 4-actuator configuration. First, sensors are assumed to be collocated with actuators. Later, additional sensors are used to improve the performance of the Kalman filter. Analyses are performed assuming that modes 1 to 6, 8 to 10, 13, 16, 20, 21, 26 and 35 are controlled. Furthermore, to obtain a valid comparison between full state feedback and output feedback control, the same control effort, $\|u\|_2$, is exerted for output feedback control as for full state feedback control.

Table VI shows a comparison of results for analyses using full state and output feedback control considering the case of lateral, vertical and longitudinal uniform-support excitation. Displayed are the reductions obtained in the 2-norms of the uncontrolled force (i.e. bending moments) response and the actuator efficiency in terms of force response reduction per unit actuator effort ($\|\mathbf{f}(t)\|_2 \div \|\mathbf{u}(t)\|_2$). The last column of the table, titled 'Output ÷ Full State Feedback', is obtained by dividing the actuator efficiencies for the output feedback case (4th column) by those of the full state feedback case (2nd column). For instance, for the case of lateral uniform-support excitation and equal control effort, output feedback control achieves 83·9 per cent of the force response reductions obtained for full state feedback control. Table VII displays the same results for the displacement response.

Table VI shows that output feedback control performs reasonably well compared to full state feedback control. As expected, full state feedback control outperforms output feedback control in every case; however, the margin is relatively small. Output feedback control performs especially well for the cases of lateral and vertical multiple-support excitation where the control achieves almost 95 per cent of the reductions obtained for full state feedback control. However, the performance penalty of using output feedback control is more obvious when considering uniform-support excitation in the lateral and longitudinal direction and multiple-support excitation in the longitudinal direction. A similar trend can be seen when considering the attenuation

Table VI. Reduction of uncontrolled force response and actuator efficiency using full state and output feedback control under uniform and multiple support excitation

Excitation type	Full state feedback: Moments $\|\mathbf{f}(t)\|_2$ (%)	Full state feedback: $\|\mathbf{f}(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Output feedback: Moments $\|\mathbf{f}(t)\|_2$ (%)	Output feedback: $\|\mathbf{f}(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Output $\div$ full state feedback* (%)
Uniform lateral	70·33	3·05	59·14	2·56	83·9
Uniform vertical	53·79	1·90	48·54	1·71	90·0
Uniform longitudinal	45·03	8·57	35·12	6·69	78·1
Multiple lateral	54·58	2·36	51·23	2·22	94·1
Multiple vertical	69·27	2·42	65·62	2·29	94·6
Multiple longitudinal	65·95	2·65	53·78	2·17	81·9

* Obtained by dividing the 4th column by the 2nd column

Table VII. Reduction of uncontrolled displacement response and actuator efficiency using full state and output feedback control under uniform and multiple support excitation

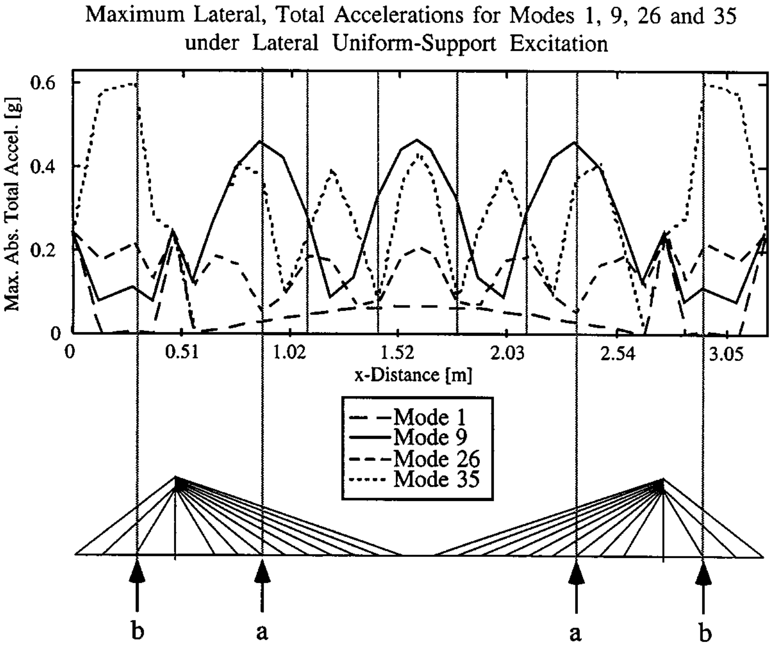
Excitation type	Full state feedback: Displ. $\|\mathbf{v}_a(t)\|_2$ (%)	Full state feedback: $\|\mathbf{v}_a(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Output feedback: Displ. $\|\mathbf{v}_a(t)\|_2$ (%)	Output feedback: $\|\mathbf{v}_a(t)\|_2 \div \|\mathbf{u}(t)\|_2$	Output $\div$ full state feedback* (%)
Uniform lateral	69·42	3·01	52·47	2·27	75·4
Uniform vertical	71·11	2·51	66·44	2·34	93·2
Uniform longitudinal	69·34	13·18	63·27	12·04	91·4
Multiple lateral	38·64	1·67	37·39	1·62	97·0
Multiple vertical	67·34	2·36	64·83	2·26	95·8
Multiple longitudinal	69·27	2·81	57·59	2·32	82·6

* Obtained by dividing the 4th column by the 2nd column

of the displacement response. However, under longitudinal uniform-support excitation, output feedback control is significantly more effective in attenuating the deck displacements than the deck bending moments. Output feedback control does not significantly increase the actuator size requirement. Peak control forces observed for output feedback control are only up to 5 per cent larger than those for full state feedback control.

In an attempt to improve the performance of output feedback control, additional sensors are distributed across the bridge, resulting in better observability of the participating modes. The main focus is on the reduction of the force response and, hence, the improvements observed in the controlled displacement response are not discussed. An attempt is made to improve the performance of output feedback control for all uniform-support excitation cases and for the case of longitudinal multiple-support excitation. The case of lateral uniform-support excitation is discussed in detail while the other cases are only summarized.

Figure 7 shows the improvements in output feedback control that are obtained through the use of additional sensor for the case of lateral uniform support excitation. Specifically, this figure displays the additional proposed sensor locations, the maximum total deck accelerations in the lateral direction for modes 1, 9, 26 and 35, and the performance of output feedback control. Since total accelerations are



Performance of Output Feedback Control

Moments: $\dfrac{(\|f(t)\|_2 \div \|u(t)\|_2)_{\text{output f.b.}}}{(\|f(t)\|_2 \div \|u(t)\|_2)_{\text{full state f.b.}}}$ Displacements: $\dfrac{(\|v(t)\|_2 \div \|u(t)\|_2)_{\text{output f.b.}}}{(\|v(t)\|_2 \div \|u(t)\|_2)_{\text{full state f.b.}}}$

Nodes Sensed	Moments	Displacements
Original	83.9%	75.4%
+ a	93.4%	91.0%
+ a + b	96.4%	95.7%

Figure 7. Performance improvements in output feedback control through the use of additional sensors: lateral uniform-support excitation

measured, sensors should be located such that the peak accelerations of the dominant modes (modes 1, 9, 26 and 35) are picked up. The four short vertical lines intersecting the acceleration lines reveal the locations of the original sensor locations, i.e. the sensors which are collocated with the actuators. The long vertical lines extending to the illustration of the bridge model show the proposed sensor locations, which are to be used in *addition* to the original four sensors. Examining the peak accelerations, it can be seen that the original sensors are placed away from the peak accelerations of mode 9, and near local minimum points for mode 35. To improve the observability of these modes, sensors at location 'a' and 'b' are proposed. Point 'a' is located where mode 9 attains maximum accelerations and where mode 35 also reaches large accelerations. Mode 35 reaches its maximum acceleration at point 'b', which is actually located on the bridge's side span. With the addition of sensors at point 'a', the performance of output feedback control is considerably improved; reductions in the force response now equal 93·4 per cent of those obtained using full state feedback control, instead of only 83·9 per cent. The performance is further increased to 96·4 per cent with additional sensors at point 'b'. This example illustrates that, as expected, the performance of output feedback control very much

depends on the sensor layout. Given a good sensor configuration, output feedback control with sensor noise can perform almost as well as full state feedback control without sensor noise.

However, the choice of additional sensor locations may not always be as obvious as the above example illustrates. More insight into the structure's dynamics is required for the case when the bridge is subject to longitudinal uniform-support excitation. Here, additional sensors located on the bridge deck did not increase the performance of the estimator significantly. Instead, additional sensors are placed on top of the towers to pick up mode 18, which is a coupled tower-deck mode that participates in the response of the deck under this excitation. Additional sensors are also proposed to improve the estimator performance for the cases of vertical uniform-support excitation and longitudinal multiple-support excitation. However, for these cases additional sensors distributed over the deck and towers only lead to small performance improvements. To obtain a significantly improved performance for these excitations, the deck needs to be supplied with a large number of sensors as there seem to be no true 'optimal' sensor locations.

In summary, the performance of output feedback control clearly depends on the number of sensors used and the sensor configuration. Generally, the more sensors that are used, the better the performance of the Kalman–Bucy filter. A thorough understanding of the structure's dynamical behaviour is required to allocate the sensors effectively. Once an effective sensor layout is chosen, output feedback control with sensor noise usually achieves approximately 90–97 per cent of the force response attenuations obtained when full state feedback control without sensor noise is considered.

ADDITIONAL ANALYSIS USING IMPERIAL VALLEY EARTHQUAKE

So far, a simplistic approach has been used to model multiple-support excitation, where the supports are assumed to move at equal amplitudes, 180° out of phase, representing one of the worst-case scenarios. In this section a more realistic representation of multiple-support excitation is considered. Here, earthquake records of synchronized, closely spaced arrays are used as input motion for the individual supports. The ground motions considered consist of Array 4, 5, 6 and 7 from the October 15, 1979 Imperial Valley (California) earthquake which were also used by Nazmy *et al.*[3] for their study on cable-stayed bridges. The earthquake is significantly larger in magnitude than the Northridge earthquake and also has significant spectral accelerations in the lower frequency regions. As a result the Imperial Valley earthquake predominantly excites the first-order modes of the bridge.

The objective of this section is to examine how a different earthquake record and a more general representation of multiple-support excitation affects the results obtained previously. Specifically, the impact of higher-order modes in the control of the bridge deck's force response is re-examined. Furthermore, some potential problems concerning the selection of the LQR weighting matrices and the estimator design are discussed.

Full state feedback control

Once again, control analyses are performed using a linear quadratic regulator considering the same weighting matrices that were obtained for the Northridge earthquake. The 4-actuator setup is used exclusively unless stated otherwise. The weighting matrices, $\mathbf{Q}$ and $\mathbf{R}$ remain unchanged from the previous analyses.

Table VIII displays the 2-norm of the attenuations in the force and displacement response obtained under lateral, vertical and longitudinal (multiple-support) excitation, as well as the maximum control force and the control effort. These results are illustrated for the case when only the first-order symmetric and antisymmetric lateral modes are controlled and for the case when higher-order modes are controlled in addition to the first-order modes.

From these results, it can be seen that the higher-order modes have much less of an impact when the bridge is subjected to the Imperial Valley earthquake as compared to the Northridge earthquake. Sufficient

Table VIII. Reduction of uncontrolled deck displacement and force response, control effort and maximum control forces using full state feedback control

	Modes controlled	1, 4	1, 4, 9, 20, 26, 35
Lateral excitation	Displacements, $\|\mathbf{v}(t)\|_2$	77·47%* (91·96%)†	77·49% (91·88%)
	Moments, $\|\mathbf{f}(t)\|_2$	89·18% (91·08%)	89·45% (91·44%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	566·60 N	575·24 N
	Peak ctrl. force	16·25 N	16·34 N
	Modes controlled	2, 3	2, 3, 6, 8, 13, 16, 21
Vertical excitation	Displacements, $\|\mathbf{v}(t)\|_2$	64·11%* (75·08%)†	63·98% (74·80%)
	Moments, $\|\mathbf{f}(t)\|_2$	56·86% (68·93%)	56·94% (69·09%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	173·34 N	175·05 N
	Peak ctrl. force	5·81 N	5·85 N
	Modes controlled	2, 3	2, 3, 5, 6, 10
Longitudinal excitation	Displacements, $\|\mathbf{v}(t)\|_2$	67·64%* (77·17%)†	67·92% (77·68%)
	Moments, $\|\mathbf{f}(t)\|_2$	70·18% (71·57%)	74·36% (76·00%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	573·91 N	587·84 N
	Peak ctrl. force	20·21 N	19·80 N

* Pseudo-static and vibrational response
† Vibrational response only

attenuation of the force response is achieved when only the first-order modes, i.e. first symmetric and antisymmetric deck modes, are controlled. Controlling the higher order modes does not yield any significant reductions. However, the control of these modes is not detrimental either, since the additional control effort exerted on these modes is minimal. The tables also show that the control effort and control forces are considerably larger when the bridge is subjected to the Imperial Valley earthquake as compared to the Northridge earthquake. This increase is a result of the different magnitudes of the two earthquakes. The more important point this increase illustrates, is that if the actuator capacity is limited, identical **Q** and **R** matrices should not necessarily be used for different earthquakes. For example, if **Q** and **R** were set such that the actuators reached their full capacities for the Northridge earthquake, a larger earthquake would most likely demand actuator forces in excess of the actuators' capacities. Similarly, if **Q** and **R** were set, such that the actuators reached their full capacities for a large earthquake, then the actuators would not be fully utilized for a smaller earthquake. To resolve this problem, a nonlinear control algorithm with adaptive **Q** and **R** matrices may be necessary.

Table VIII also shows that the multiple-support excitation effects are most prevalent under vertical support excitation. For this case, the attenuations achieved in the vibrational forces are considerably larger than those achieved in the total forces (vibrational and pseudo-static). Since pseudo-static forces are not controlled, the difference in the attenuations of the vibrational and total response can be attributed to multiple-support excitation effects. As Table VIII illustrates, this more generalized representation of multiple-support excitation does not reveal any new findings from those obtained for the simplified representation.

Output feedback control

The Kalman–Bucy filter developed for the Northridge earthquake is utilized to examine the effectiveness of output feedback control using the Imperial Valley earthquake. The implementation of the Kalman–Bucy only depends on the selection of the sensor and excitation noise intensities. Since the previous sensors with

a range of $\pm 2g$ are employed, the sensor noise intensities remain unchanged. However, excitation noise intensities do not remain constant since the Imperial Valley earthquake is of larger magnitude than the Northridge earthquake. Furthermore, the ground motion at the individual supports are correlated differently. Since Kalman–Bucy filters are not adaptive to different earthquakes, the question needs to be addressed as to how well the original estimator (designed for the Northridge earthquake) performs for the Imperial Valley earthquake. Control analyses are performed employing the original estimator and a modified estimator specifically designed for the Imperial Valley earthquake. Table IX shows the results of control analyses employing the original and the modified Kalman–Bucy filter under lateral, vertical and longitudinal support excitation. Illustrated are the reductions obtained in the 2-norm of the control effort, displacement and force response, and the maximum control effort. In addition, to show the effectiveness of the output feedback control relative to full state feedback control, the table shows how close the reductions obtained for output feedback control come to those for full state feedback control. The results are obtained assuming a total of 8 sensors (4 in both vertical and lateral directions), which are collocated with the actuators. Modes 1 to 6, 9, 10, 13, 16, 21, 26 and 35 are assumed to be controlled.

Table IX shows that although the performance of output feedback control with the original Kalman–Bucy filter is adequate, the performance is significantly improved when the Kalman–Bucy filter is adjusted for the new earthquake. These improvements can be seen for the case of lateral, vertical and longitudinal excitation. The inconsistent performance of the different estimators addresses a similar problem to the one observed in the linear quadratic regulator design. The problem lies in that the estimator performance depends on the choice of earthquake records which are used for the estimator design. Thus, it is important that the estimator is based on ground motions that are representative of the bridge location. Furthermore, the estimator should be tuned so that it performs well for a large number of different earthquakes or should be adaptive to the earthquake.

The effectiveness of the Kalman–Bucy filter also depends on the number of sensors and their locations. Table IX assumed that all sensors are collocated with the actuators. Additional sensors were proposed to

Table IX. Performance of output feedback control using original and modified estimators for lateral, vertical, and longitudinal excitation. Reduction of uncontrolled displacement and force response, control effort, maximum control force

	Estimator	Original	Modified
Lateral excitation	Displacements, $\|\mathbf{v}(t)\|_2$	72·15%* (93·11%)†	75·36% (97·25%)
	Moments, $\|\mathbf{f}(t)\|_2$	77·68% (86·84%)	83·46% (93·30%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	575·10 N	575·01 N
	Peak ctrl. force	14·81 N	15·93 N
	Estimator	Original	Modified
Vertical excitation	Displacements, $\|\mathbf{v}(t)\|_2$	56·68%* (88·59%)†	61·17% (95·61%)
	Moments, $\|\mathbf{f}(t)\|_2$	49·36% (86·69%)	54·30% (95·36%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	174·96 N	175·01 N
	Peak ctrl. force	5·22 N	5·49 N
	Estimator	Original	Modified
Longitudinal excitation	Displacements, $\|\mathbf{v}(t)\|_2$	59·69%* (87·88%)†	64·47% (94·92%)
	Moments, $\|\mathbf{f}(t)\|_2$	63·30% (85·13%)	70·60% (94·94%)
	Ctrl. effort, $\|\mathbf{u}(t)\|_2$	586·71 N	588·74 N
	Peak ctrl. force	18·59 N	19·40 N

* Reduction of combined pseudo-static and vibrational response

† Performance of output feedback control (attenuation for output feedback control ÷ attenuation for full state feedback control)

increase the performance of the estimator when subject to the Northridge earthquake. However, the modified estimator without the additional sensor already performs very well when subjected to the Imperial Valley earthquake. The additional sensors do not result in an improved performance since the original 8 sensors already performed well. The Imperial Valley earthquake predominantly excites first order modes, which are easily picked up by the original sensor configuration; hence, the good performance of the original sensor configuration.[1]

SUMMARY AND CONCLUSIONS

Using a 42 state reduced-order model, control analyses are performed assuming full state and output feedback control with sensor noise utilizing a linear quadratic regulator and a Kalman–Bucy filter. The focus of the control is on the reduction of the deck's force response. Simulations are performed considering the 1994 Northridge earthquake and the 1979 Imperial Valley earthquake.

This study concludes that the linear quadratic regulator is very effective in reducing the deck's force and displacement response. However, the force response is not automatically controlled when the displacement response is controlled, especially for ground motions with a high frequency content. Although the force response is controlled to some extent for this case, significantly higher attenuations are achieved when the emphasis is on the force response. Generally, higher- and lower-order modes need to be controlled to attenuate the force response, while only lower-order modes need to be controlled to attenuate the displacement response. The impact of the higher-order modes is more prevalent for the case of uniform-support excitation. Multiple-support excitation typically excites different modes than uniform-support excitation. Naturally, these modes are excited if their frequencies are close to the earthquake's dominant frequencies. Thus under multiple-support excitation (and uniform-support excitation), the impact of the higher-order modes on control depends on the earthquake.

Analyses performed using output feedback control show similar results to those found for state feedback control. Attenuations in the force response equal to approximately 90–97 per cent of those obtained for full state feedback control can be achieved. Sensors measuring total accelerations are assumed to be collocated with the actuators. To improve the performance of the estimator, additional sensors are placed on the deck and towers to pick up the peak accelerations of the dominant modes.

Results also show that only few actuators are required to control the bridge and that actuator locations close to the center of the bridge deck are most effective. Very efficient control is achieved using 3 actuators in both vertical and lateral directions. However, the majority of this research is conducted using 4 actuators in the vertical and lateral directions, since considerably smaller actuators are required for this case than for the 3-actuator configuration.

One of the most significant findings of this study is that not all modes can be controlled. Using actuators located on the bridge deck, the attempt of controlling a coupled tower-deck mode (mode 18) resulted in a decrease of the towers' force response, but also in a significant increase of the deck's force response and control effort. Hence, this mode should not be controlled regardless of the ground motion acting on the bridge. Coupled mode shapes are not uncommon for cable-stayed bridges. These modes not only increase the complexity of the bridge's vibrational behaviour, but also the complexity of active control and, thus, *must* be addressed when considering active control for cable-stayed bridges.

The majority of control analyses are performed considering the 1994 Northridge earthquake. In contrast to the Northridge earthquake, the 1979 Imperial Valley earthquake is of larger magnitude and has a lower frequency content resulting in a larger force response dominated by low-order modes. Results show that very large control forces develop for the case when the regulator designed for the Northridge earthquake is used for the Imperial Valley earthquake. Furthermore, in the case of output feedback control, the estimator performs considerably better when it is tuned to a specific ground motion. Therefore, care must be taken to ensure that the regulator and estimator perform well for a large number of earthquakes that are representative of the bridge site.

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